

INDUCTION COILS

A THEORETICAL AND EXPERIMENTAL RESEARCH

BY

BENJAMIN F. BAILEY

Assistant Professor of Electrical Engineering in the University of Michigan

A THESIS

SUBMITTED TO THE FACULTY OF THE DEPARTMENT OF LITERATURE,
SCIENCE AND THE ARTS OF THE UNIVERSITY OF MICHIGAN,
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY,
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INDUCTION COILS.

BY BENJ. F. BAILEY.

MY object in this investigation was first and principally to establish the conditions of maximum efficiency in primary coils, and to deduce a method whereby the efficiency of such coils could be predetermined from their electrical design; secondly to verify experimentally this theory; and thirdly to study the ordinary induction coil with secondary current from both theoretical and experimental standpoints. The efficiency of the secondary coil was also investigated and determined experimentally in the case of one coil.

By an induction coil I mean an apparatus in which magnetic energy is stored, to be used later in some different form from that in which it was supplied. Ordinarily, it is also to be understood that the charging of the coil is to be accomplished from a constant potential source, and usually, the intention is to utilize the stored energy at a higher potential and at a greater wattage than that at which it was supplied. Such coils are made in a great variety of forms. The simplest is a plain coil of one or more turns of a non-magnetic conductor, and an arrangement for making and breaking the circuit. Such an apparatus will not store much energy per unit of weight and consequently can be used economically only when it is charged and discharged a great many times per second. An example of this is the Tesla high frequency coil. For ordinary purposes the introduction of an iron core greatly increases the capacity of the coil. The voltage obtainable from such a coil is still rather limited, and if higher values are desired, they may be obtained by winding a secondary coil of a greater number of turns around the primary coil. For most purposes this arrangement is improved by the use of a condenser connected across the break. These various forms of the apparatus will be taken up in the order described.

PRIMARY OR SINGLE CIRCUIT COILS.

The primary coil, while a relatively unimportant device, is used in the ignition of almost all stationary gas engines and with many boat and automobile engines. Moreover, the theory of these coils as developed here will be used directly in considering the case of the double circuit or ordinary induction coil. This, together with the fact that so far as I know, nothing has been written regarding the theory of the primary coil, is the reason for my taking it up here. The first point to be considered is the efficiency of such coils.

EFFICIENCY OF PRIMARY COIL.

By the efficiency of any piece of electrical apparatus we mean the ratio of the useful work obtained from it to the total work put into it. In obtaining this for any piece of apparatus two methods are in general available. The most obvious one is to measure these quantities directly. The case of the simple primary circuit is shown in

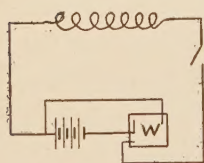


Fig. 1.

Fig. 1. To apply this method we might use a wattmeter as indicated to measure the input. Theoretically, the output might be measured by transferring the connection *a*, from the terminal of the battery to the other side of the break at *b*. Practically, this would not work well for several reasons. For one thing the high voltage at the break would be liable to puncture the insulation of the wattmeter. Moreover, the amount of power required to operate the wattmeter is large compared to the power of the spark; or stated differently, a large part of the current would shunt off through the wattmeter instead of passing across the gap. This would obviously affect the intensity of the spark, making it much less than it should be. Also since the current and E.M.F. are far out of phase, the self-induction of the wattmeter coils would cause the results to be in error to a considerable extent.

Another method of accomplishing the same object would be to measure the heat produced by the spark by means of a small calorimeter. This method is difficult to apply and is not very accurate.

The method here deduced is in principle the same as that used to predetermine the efficiency of all large generators, transformers, etc.

It is based upon the principle of the conservation of energy, *i. e.*, all the energy that goes into the coil must appear in some form or other. Part appears as the energy of the spark and the rest is wasted in various losses. Stated mathematically we have

$$\text{Eff.} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

This method is comparatively easy to apply to the case of a dynamo or transformer. With a spark coil it is more difficult, as the calculations are more involved owing to the shape of the waves used. It should be remarked that while the method appears indirect at first, it is in reality preferable to the direct method, both on the score of accuracy, and even more because it shows the magnitude of the various losses, and thus indicates how the design of the coil may be modified so as to improve it.

COIL WITHOUT IRON CORE. INSTANTANEOUS BREAK.

We will first take up the case of a coil without an iron core. This is the simplest case as there are no iron losses to consider. In a circuit like that of Fig. 1 in which the total resistance of the circuit is R and the inductance is L , the differential equation is:

$$E = Ri + L \frac{di}{dt}$$

and

$$i = I(1 - e^{-Rt/L}).$$

The magnetic energy stored in the coil at any instant is $\frac{1}{2}Li^2$. If the circuit is broken instantaneously, all of this energy (with the possible exception of a small loss due to eddy currents in the copper) appears as the energy of the spark. This will be practically all heat energy, but a small proportion may be radiated as electromagnetic waves. It is here assumed that all the energy of the spark is used, and the output is consequently $\frac{1}{2}Li^2$.

The input will be the output plus the losses. The latter here consist merely of the ohmic loss in the winding, during the process of building up the magnetism. To get this we must find

$$\int_0^T Ri^2 dt.$$

R is a constant and substituting for i we get

$$\begin{aligned} R \int_0^T i^2 dt &= R \int_0^T I^2 \left(1 - \epsilon^{-\frac{Rt}{L}} \right)^2 dt \\ &= RI^2 \int_0^T \left(1 - 2\epsilon^{-\frac{Rt}{L}} + \epsilon^{-\frac{2Rt}{L}} \right) dt. \end{aligned}$$

Integrating this we get

$$RI^2 \left[T + \frac{L}{2R} \left(1 - \epsilon^{-\frac{2RT}{L}} \right) - \frac{2L}{R} \left(1 - \epsilon^{-\frac{RT}{L}} \right) \right] = Om.$$

The total input is

$$\frac{1}{2} Li^2 + Om = \frac{1}{2} Li^2 + RI^2 T + \frac{1}{2} LI^2 (1 - \epsilon^{-2RT/L}) - 2LIi$$

which readily reduces to $RI^2 T - LIi$. The efficiency then becomes

$$\eta = \frac{\frac{1}{2} Li^2}{RI^2 T - LIi}.$$

This gives the efficiency in terms of L , i , R , I , and T . It is obvious that these quantities are interconnected, and by eliminating we should arrive at a simpler form. To eliminate T take logarithms of both sides of

$$\frac{I-i}{I} = \epsilon^{-\frac{RT}{L}}$$

and get

$$T = \frac{L}{R} \log \frac{I}{I-i}.$$

Substituting this in the above we obtain

$$\eta = \frac{\frac{1}{2} i^2}{I^2 \log \frac{I}{I-i} - Li}.$$

This gives us at once the very peculiar results that the efficiency is independent of both the resistance and the inductance and depends only on the value of the current at the instant of break and upon its steady value. This may be made clearer and some new laws brought out by a further change.

To do this substitute

$$\log \frac{I}{I-i} = \left(\frac{i}{I} \right) + \frac{1}{2} \left(\frac{i}{I} \right)^2 + \frac{1}{3} \left(\frac{i}{I} \right)^3 + \dots,$$

simplify and get

$$\eta = \frac{\frac{1}{2} \left(\frac{i}{I} \right)^2}{\frac{1}{2} \left(\frac{i}{I} \right)^2 + \frac{1}{3} \left(\frac{i}{I} \right)^3 + \frac{1}{4} \left(\frac{i}{I} \right)^4 + \dots}.$$

Or if we designate the ratio i/I by a we obtain

$$\eta = \frac{\frac{1}{2} a^2}{\frac{1}{2} a^2 + \frac{1}{3} a^3 + \frac{1}{4} a^4 + \dots}.$$

From this very simple form we can at once deduce several important conclusions.

I. The efficiency is entirely independent of the ohmic resistance of the coil, for a given ratio of i/I .

II. The efficiency is entirely independent of the inductance of the coil.

III. The efficiency is a function of the ratio of the current at break, to the current which would flow if the contact were very long.

IV. The efficiency is a maximum and equal to 100 per cent. for $i = 0$.

V. The efficiency is a maximum and equal to 100 per cent. for $I = \infty$.

We may write the expression for the efficiency as follows :

$$\eta = \frac{\frac{1}{2} \left(\frac{iR}{E} \right)^2}{\frac{1}{2} \left(\frac{iR}{E} \right)^2 + \frac{1}{3} \left(\frac{LR}{E} \right)^3 + \frac{1}{4} \left(\frac{iR}{E} \right)^4 + \dots}.$$

We then conclude in addition to the above

VI. With given values of R and i the efficiency is a maximum and equals 100 per cent. for $E = \infty$.

VII. With fixed values of E , i and L the efficiency is a maximum and equals 100 per cent. for $R = 0$.

This latter condition is the one present in practice. E is fixed, at least to a certain extent, by the cost and the difficulty of carrying many cells in vehicles or boats. L being also fixed, i must not be less than a certain value or the resultant spark will not be sufficiently intense to ignite the charge. Under these conditions R should be as small as possible with the given inductance.

The above facts can also all be proved by direct differentiation of the formula

$$\eta = \frac{\frac{1}{2}i^2}{I^2 \log \frac{I}{I-i} - iI}$$

So far, we have neglected entirely the influence of the time of break. It is evident that during this time an additional amount of energy is drawn from the battery. Part of this appears as heat in the spark, and the rest is wasted in I^2R loss in the resistance of the coil.

To calculate the magnitude of this effect, it is necessary to know something about the shape of the current curve during the time that the circuit is being opened. To deduce mathematically the shape of this curve would be difficult if not impossible. It will be seen, however, from the accompanying curves that it is practically a straight line. Assuming this to be the case, the current may be expressed by $i = K(T - t)$ in which T is the time that elapses from the beginning of opening of the circuit until the resistance becomes infinite and t is the time. Since at $t = 0$, $i = i_b$, $K = i_b/T$ or $i = i_b/T(T - t)$.

We may now readily modify our expression for efficiency so as to include this factor. The formula

$$\eta = \frac{\frac{1}{2}\left(\frac{i}{T}\right)^2}{\frac{1}{2}\left(\frac{i}{T}\right)^2 + \frac{1}{3}\left(\frac{i}{T}\right)^3 + \frac{1}{4}\left(\frac{i}{T}\right)^4 + \dots}$$

may be written

$$\eta = \frac{\frac{1}{2}Li^2}{\frac{1}{2}Li^2 + LI^2 \left[\frac{1}{3}\left(\frac{i}{T}\right) + \frac{1}{4}\left(\frac{i}{T}\right)^4 + \dots \right]}$$

Adding $\frac{1}{2}Ei_bT$ to both members, subtracting $\frac{1}{3}Ri_b^2T$ from the numerator and dropping the subscript in i_b we get

$$\eta = \frac{\frac{1}{2}Li^2 + \frac{1}{2}EiT - \frac{1}{3}Ri^2T}{\frac{1}{2}Li^2 + \frac{1}{2}EiT + LI^2 \left[\frac{1}{3}\left(\frac{i}{T}\right) + \frac{1}{4}\left(\frac{i}{T}\right)^4 + \dots \right]}$$

EFFECT OF TIME OF BREAK.

To find the influence of the time of break upon the efficiency we differentiate with respect to T and place the result equal to zero. For this purpose we may regard everything except T and η as constant and write

$$\eta = \frac{K_1 + K_2 T}{K_3 + K_4 T}.$$

Then

$$\frac{d\eta}{dT} = \frac{K_2 K_3 - K_1 K_4}{(K_3 + K_4 T)^2}.$$

This is satisfied for $T = \infty$ and in this case we have

$$\eta = \frac{K_2}{K_4} = \frac{\frac{1}{2}Ei - \frac{1}{3}Ri^2}{\frac{1}{2}Ei} = 1 - \frac{2}{3}\alpha.$$

The efficiency with $T = 0$ was

$$\eta = \frac{\frac{1}{2}\alpha^2}{\frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 + \dots} = 1 - \frac{2}{3}\alpha - \frac{1}{18}\alpha^2 - \frac{2}{135}\alpha^3 - \frac{4}{620}\alpha^4 \dots$$

The first two terms of these expressions are the same, and they differ only by small multiples of α^2 , α^3 , $\dots$, etc. Hence, when the ratio $\alpha = i/I$ is small, the efficiency is very little affected by the speed of break. In any case however it becomes somewhat greater as the time of break becomes larger.

THE SPARK COIL WITH IRON CORE.

The coil without iron, while it may have high efficiency as has been shown, is probably impracticable for ordinary purposes. Almost the only use to which primary coils are put, is the ignition of gas engines, for gas lighters, and similar uses. Experience shows that for such work an output of about 60×10^{-4} joules is required. If i be taken as 1 ampere, which is about as much as we would care to use from the ordinary primary battery, we have, assuming 100 per cent. efficiency, $\frac{1}{2}Li^2 = 60 \times 10^{-4}$ or $L = 1.0120/1 = .012$ henry. A coil without an iron core, to have this inductance, would necessarily be of very large size. This is true since it would have to be wound with coarse wire to keep the ratio i/I small when used with a moderate voltage.

The introduction of an iron core gives us three additional losses

to compute. These are the hysteresis loss, eddy current losses at make and at break. We shall designate these respectively by H , E_m and E_b . The expression for efficiency then becomes

$$\eta = \frac{\frac{1}{2}Li^2 - E_b + I_b}{\frac{1}{2}Li^2 + E_m + O_m + H + O_b + I_b},$$

we have now to find expressions for H , E_m and E_b .

The hysteresis loss is best found by direct experiment. In the coil here considered three methods have been employed. One is to use a wattmeter and alternating currents, the other consists in obtaining the hysteresis curve or loop for the coil in question and from its area, as from an indicator card, obtaining the actual loss in one cycle. A third method is to compute the loss from the average values obtained from similar samples of iron and used by dynamo designers and others to predetermine hysteresis loss. The three methods give very concordant results. It is well known that this loss for each cycle varies as the 1.6 power of the flux density. The flux density for small values is proportional to the current, consequently we may write $H = \text{constant} \times i^{1.6}$. In the coil considered the constant is equal to 4.69×10^{-4} or $H = 4.69 \times 10^{-4} i^{1.6}$.

The loss due to eddy currents at the make has next to be determined. As the magnetic flux surges back and forth through the wire, currents, first in one direction, then in the other, will be generated and will flow in circles, through the wire. This current wastes power in heat, and, though the amount is small it is not negligible.

Consider a ring of radius r and thickness dr . If φ is the flux through this ring, the E.M.F. induced in it is $e = d\varphi/dt$. But we may write $\varphi = Kai$, where K is a constant depending upon the coil used and a is the area of the ring. Then

$$e = Ka \frac{di}{dt} = \pi r^2 K \frac{di}{dt}.$$

But

$$i = I(1 - e^{-Rt/L}),$$

and

$$\frac{di}{dt} = I \frac{R}{L} e^{-\frac{Rt}{L}}.$$

Substituting and dividing by 10^{-8} to reduce to volts, we get

$$e = 10^{-8} K \pi r^2 I \frac{R}{L} \epsilon^{-\frac{Rt}{L}}.$$

The resistance of the cylinder of which this ring is an element and of a length equal to the length of the coil, say l is equal to

$$\text{Constant} \times \frac{\text{length}}{\text{cross-section}} = \frac{K_1 2\pi r}{ed r}.$$

The power loss at any instant is

$$\frac{e^2 dr}{r} = \frac{K^2 \pi r^3 I^2 R^2 \epsilon^{-2Rt/L}}{10^{16} \times 2K_1 L^2} dr.$$

There are only two variables in the equation, t and r , and to find the rate of loss of energy in one wire of the core we must integrate with respect to r from 0 to $D/2$, where D is the diameter of the wire. Doing this, we get

$$w = \frac{K^2 \pi I^2 R^2 \epsilon^{-2Rt/L}}{10^{16} \times 2K_1 L^2} \int_0^{D/2} r^3 dr = \frac{K^2 \pi I^2 R^2 l}{10^{16} \times 128 L^2 K_1} \epsilon^{-\frac{2Rt}{L}} D^4.$$

To get the loss of energy in one cycle we must multiply this by dt and integrate again from 0 to T_m , where T_m is the time elapsing from the make to the break. This gives us

$$j = \frac{K^2 \pi I^2 R^2 l D^4}{10^{16} \times 128 K_1 L^2} \int_0^{T_m} \epsilon^{-\frac{2Rt}{L}} dt = \frac{K^2 \pi I^2 R l D^4}{10^{16} \times 256 L K_1} (\epsilon^{-\frac{2RT_m}{L}} - 1).$$

which may be readily simplified to

$$\frac{1.36 K^2 R l D^4}{10^{13} L} i(2I - i).$$

We can easily show that

$$K = \frac{10^8 L}{NA},$$

where N is the number of turns of wire in the coil and A is the net area in square centimeters of the core. Then we have finally, for the whole coil, w being the number of wires in the core, loss per cycle due to eddy currents during make in joules equals

$$E_m = \frac{1.360 L R l D^4 W}{N^2 A^2} i(2I - i).$$

This is true for any coil, and if the value of L, R, l, D, W, N and A in any particular case be inserted it reduces to the form constant $\times i(2I - i)$. In the case of the coil here considered, it becomes $7 \times 10^6 i(2I - i)$.

To find the losses at the break we must know something about the way in which the current falls off during this time. Assuming as before that the current during break follows a straight line law we may take for our expression for the current $i = -K_3 t$, taking a new origin for the time, for the sake of simplicity.

To get the eddy current loss at break we proceed as before, merely using our new value for i . Thus

$$e = \frac{d\theta}{dt}; \quad \theta = -Kai = -KaK_3 t;$$

then $e = -KK_3 a = -\pi r^2 KK_3$. Taking r as before,

$$dw = \frac{e^2}{r} = \frac{\pi r^3 K^2 K_3^2 l}{2K_1} dr.$$

Then rate of loss of energy per wire per cycle equals

$$W = \frac{\pi K^2 K_3^2 l}{2K_1} \int_0^{D/2} r^3 dr = \frac{\pi K^2 K_3^2 l D_1^4}{128 K_1},$$

and the energy loss in joules per cycle for the whole core of W wires, if T_b be the time of break, is

$$E_b = \int_0^{T_b} W dt = \frac{\pi K^2 K_3^2 l T_b D_1^4 W}{128 K_1}.$$

If i is the value of the current at the instant of break, then

$$i = -K_3 T_b \quad \text{or} \quad K_3 = -\frac{i}{T_b} \quad \text{and} \quad K = \frac{L}{NA}.$$

Substituting also for K_1 its value for iron of 9×10^{-6} we get

$$E_b = \frac{\pi L^2 i^2 l T_b D^4 W}{128 N^2 A^2 T_b^2 \times 9 \times 10^{-6}} = \frac{\pi L^2 l D^4 10^3}{1.152 N^2 A^2} \times \frac{i^2}{T_b} \times W.$$

In the coil considered this reduces to

$$7.85 \times 10^{-7} \frac{i^2}{T_b}.$$

The loss of energy in the coil during the break is next to be obtained. As before, $i = -K_3 t$. The loss of energy equals

$$\int_0^{T_b} i^2 R dt = K_3^2 R \int_0^{T_b} t^2 dt = \frac{1}{3} K_3^2 R T_b^3.$$

If, as before, i = current at time of break, then $i = -K_3 T_b$ and loss in joules per cycle = $O_b = \frac{1}{3} R i^2 T_b$. In the coil here considered $R = 0.725$ ohm, and we get $O_b = 0.242 i^2 T_b$.

We have to obtain now only the added energy furnished by the battery during the break. Since we have assumed that the current varies uniformly from a value i to zero during this time, the average power is evidently $\frac{1}{2} Ei$. Then the energy added equals

$$I_b = \frac{1}{2} Ei T_b.$$

This gives us all the factors needed for the expression for efficiency. As before

$$\eta = \frac{\frac{1}{2} Li^2 + I_b - E_b - O_b}{\frac{1}{2} Li^2 + I_b + H + E_m + O_m}.$$

Substituting the values we have just found for these factors, we obtain a general expression for the efficiency of any primary coil. Substituting the values of L, R, l, D, N, A, K_4 and W for the coil under consideration the formula will be greatly simplified. In the case of the coil here considered it becomes

$$\eta = \frac{0.02 i^2 + \frac{1}{2} Ei T_b - 0.242 i^2 T_b - 7.85 \times 10^{-7} \frac{i^2}{T_b}}{0.02 i^2 + \frac{1}{2} Ei T_b + 4.69 \times 10^{-4} i^{1.6} + 7 \times 10^{-6} i (2I - i)} + 0.04 I^2 \left[\frac{1}{3} \left(\frac{i}{I} \right)^3 + \frac{1}{4} \left(\frac{i}{I} \right)^4 + \dots \right].$$

To apply this we must know the time of make and break. The former is most readily found by turning the engine over slowly by hand and observing the angle within which contact is made. Knowing the number of revolutions per minute we can easily compute the time of contact. The time of break cannot be obtained in any simple way. It will usually be about one one-thousandth of a second, however, and any small error in estimating it will have but very little influence upon the result. In the case of one engine it

was found that the time of contact was one one-hundredth of a second. Knowing this and the E.M.F. of the battery used, which was in this instance 2.52 volts, we can calculate the value of i .

Substituting in $i = I(1 - e^{-Rt/L})$ we get

$$i = \frac{2.52}{0.725} \left(1 - e^{-\frac{0.725 \times \frac{1}{100}}{.04}} \right) = 0.584 \text{ ampere}$$

and

$$I = \frac{E}{R} = \frac{2.52}{0.725} = 3.5 \text{ amperes.}$$

Substituting these values in the formula we find the efficiency equal to 82.5 per cent. That is, of the actual energy put into the coil 82.5 per cent. appears in the heat of the spark, the other 17.5 per cent. being wasted in the various losses.

It is of some interest to compare these losses with one another and with the input. They are given in the following table :

Magnetic energy,	$\frac{1}{2} Li^2 = 68.4 \times 10^{-4} \text{ joules}$
Energy added during break,	$Ib = 7.35 \times 10^{-4} \text{ joules}$
Ohmic loss at make,	$Om = 8.80 \times 10^{-4} \text{ joules}$
Ohmic loss at break,	$Ob = 0.83 \times 10^{-4} \text{ joules}$
Eddy current loss at make,	$Em = 0.25 \times 10^{-4} \text{ joules}$
Eddy current loss at break,	$Eb = 2.68 \times 10^{-4} \text{ joules}$
Hysteresis loss,	$H = 2.03 \times 10^{-4} \text{ joules}$

Of these the largest is the ohmic loss at the make. This can be reduced by using a battery of higher E.M.F. and a correspondingly short make. This is readily seen to be true, since this loss is dependent upon the ratio of i/I , and increasing the E.M.F. increases I in the same proportion. The ohmic loss at the break is small in any case, and can be still further reduced by shortening the time of break.

In the eddy current losses we have an opposite set of conditions, since each is increased by a decrease of the time of make or break. Hence the conditions for best efficiency here are the exact opposite to those applicable to the ohmic losses. Fortunately, the eddy current loss can be reduced to any desired extent by using finer wires in the core. For a given cross-section of core the loss is in proportion to the square of the diameter. It will be noticed that the loss is, of course, much larger at the break than at the make. Hence it has an especially bad effect, as it takes away materially

from the energy of the spark. Indeed, a short calculation shows that if the break takes place in about thirty-five millionths of a second all the energy would be wasted in the eddy currents.

The magnitude of the hysteresis loss is independent of the time of the make or break, and depends only on the value of the current used and the quality of the iron. To reduce it as far as possible, iron having the smallest hysteresis loss possible should be used.

In any piece of apparatus the efficiency curve is of importance; that is, it is important to see whether the efficiency keeps up well through a wide range of load. I have computed the efficiency for various values of the current i , that is, the current at which the break takes place, the results being given in Table I. The values of i correspond to an intensity of spark from 0 to 0.04 joule. The actual value of i used, it will be remembered, is 0.584 ampere, corresponding to 0.0075 joule per spark. At $i = I = 2.5$ amperes the efficiency would drop to zero.

TABLE I.

Efficiency for Different Values of i .

i	Efficiency.
0	77 per cent.
0.1	83 "
0.2	85 "
0.4	83 "
1.0	81 "

The fact that the efficiency stays above 80 per cent. up to about five times the spark intensity used shows that the coil is capable of delivering a spark five times as strong as actually used and still preserve a good efficiency, or, conversely, a very much smaller coil would be capable of doing the work required. At the same time a slightly greater efficiency could be obtained by an even larger coil. However, the gain would be slight and it would seem better to reduce considerably the size of the coils usually used.

To summarize—for the highest efficiency the battery should have a rather high E.M.F. and the contact should be short, so that the current rises only to a small fraction of the value it would attain if given unlimited time. The break should be reasonably short, but not too brief, especially with a coil having wire of a large diameter in the core, and the core should be made of wire having the

smallest possible hysteretic coefficient. For the sake of first cost, the coil could be much smaller than those usually used.

EXPERIMENTAL WORK WITH THE PRIMARY INDUCTION COIL.

The preceding theory is, it is believed entirely new, and it seemed desirable to confirm it by direct experimental proof. The method adopted is equally applicable to the ordinary induction coil with a secondary winding, and was used in obtaining various results in connection with such coils.

The first point taken up was an experimental determination of the efficiency of a primary coil. The method adopted was to obtain by means of an instantaneous contact maker, a series of points on the curves of current and E.M.F. of the coil. The summation of the products of E.M.F. and current will give us the energy input and output and consequently the efficiency, and in addition we learn the way the current rises, how it falls off, the inductive rise of pressure at the break, both across the gap and over the coil, the potential difference of the battery, etc. These are gotten directly and having them we may easily compute the values for the curves of power during make and break, the resistance or conductivity of the gap during break, and the internal resistance of the battery.

Some difficulty in obtaining points especially during the break was anticipated. Looking at the sparks, they appear to differ considerably in length, quality, etc., and hence it was feared that it would be impossible to secure consistent results. Some trouble was encountered from this source, but by keeping the contacts clean, this was reduced to a minimum.

The use of the oscillograph to obtain these curves has been suggested, and for merely qualitative results it could undoubtedly be employed to very great advantage. For accurate work, however, it could hardly be used with spark coils. For one thing it would take a relatively large amount of energy from the circuit, especially during the break. For example, the oscillograph manufactured by the General Electric Company requires 0.3 ampere for its operation. At the break the voltage rises to from 50 to 100 volts, and consequently from 15 to 30 watts would be expended in the instrument. Since the maximum wattage of the spark rarely exceeds twice these

figures, it is evident that this method is out of the question for the determination of the efficiency. This objection does not apply to the oscillograph when used on circuits carrying considerable amounts of power.

The arrangement of the apparatus used is shown in Fig. 2. The main circuit is shown in the upper part of the figure and comprises from left to right a battery B , a switch S , a rotating break Br , a

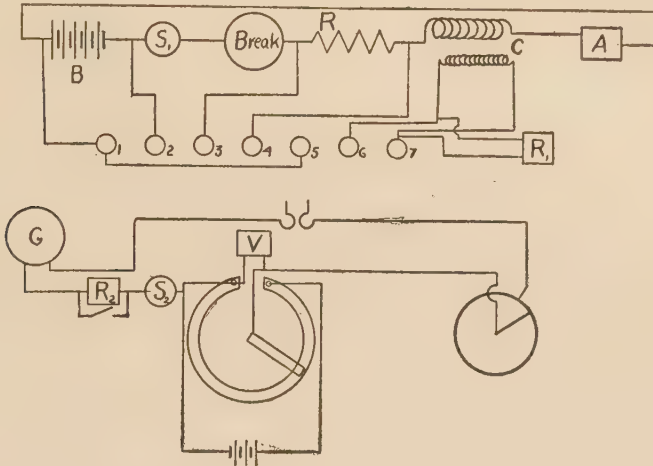


Fig. 2.

non-inductive resistance R of low value, the coil under test C , here shown with a secondary, and an ammeter A . This is the usual circuit of a primary coil as used in gas engine ignition, the only difference being the use of a rotating break and the insertion of the extra resistance R . The former seemed preferable from the standpoint of uniformity of action, and the latter is necessary to obtain the curve of current. It has a resistance, however, of only 0.133 ohm, and in fact amounts to no more than the resistance of the usual wiring of a coil. The fact that we are here operating under actual working conditions is very important.

From points on the circuit as shown, connections are made to a row of mercury cups. By means of the fork-shaped arrangement shown just below the mercury cups, connection can be made to any part of the circuit. Thus taking them in order from left to right, we can connect with the battery, the break, the non-inductive

resistance, the primary of the coil and the secondary of the same. With the primary is included the ammeter, which may be considered an extra inductance and resistance added to the coil. Its influence, however, is negligible.

The contact maker marked *C.M.* is of the usual form used in investigating E.M.F. waves. It is rigidly connected to the shaft of the rotating break and the whole is operated by a 1 H.P. motor. The current for the motor is taken from a storage battery. Thus the pressure and hence the rate of rotation are very uniform. Changes of speed are provided for by varying the number of cells connected to the armature. The field is kept constant. In this arrangement no resistance is used, hence the rotation at low speeds is much more uniform than would otherwise be possible.

S.R. (Fig. 2) is a shunt rheostat. This is used to give any desired potential difference from zero to that of the battery employed. In the actual setup, a slightly more complicated arrangement was used. It was necessary to have a variable voltage ranging from zero to about one hundred volts, and instruments capable of reading accurately any voltage in this range. This was accomplished by using two voltmeters, each having two scales. One was a milli-voltmeter giving a full scale deflection with 0.03 or 0.3 volt depending upon which scale was used. The other voltmeter had ranges of 3 and of 150 volts. Any one of these four scales could be readily connected as desired. For convenience in getting low voltages, two batteries instead of one as shown were used and either could be readily thrown in as desired. For a fine adjustment, a low adjustable resistance was used in series with the battery. This is also omitted in the figure. The instantaneous voltage across any part of the circuit can be balanced against the voltage at the terminals of the shunt rheostat, and the curves thus determined point by point. Current curves are obtained by measuring the voltages over the non-inductive resistance and dividing by the resistance.

This method while rather laborious, has four very important advantages over any other. The first is that the circuit is working under absolutely normal conditions, the resistance of *R* and that of the ammeter being no more than would usually be present in

practice. Secondly the voltages are read directly and on ordinary voltmeters. Therefore the accuracy is great, no computation is required and the chance of error is slight.

The third and greatest advantage is that absolutely no current is taken from the circuit in making the measurements. This follows of course from the fact that this is a balance method, the energy to operate the voltmeter being taken from the auxiliary battery. This is very important as the amount of energy carried is small and if any were required to operate the instruments, the error would be considerable.

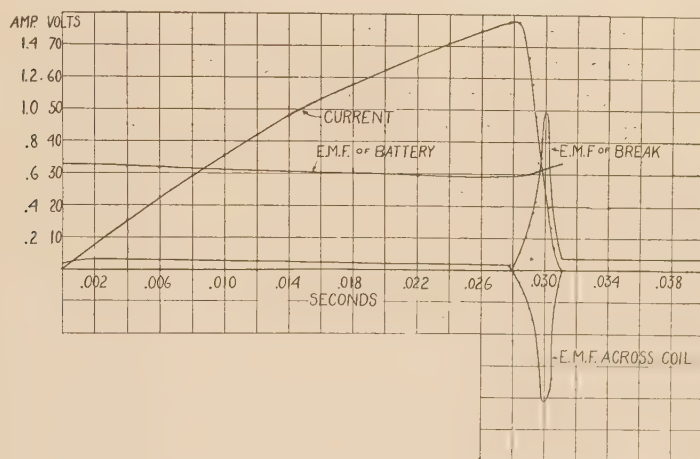


Fig. 3.

It should also be pointed out that changes in the force with which the brush of the contact maker presses on the rotating steel piece, do not affect the accuracy of the method. A poor contact has simply the effect of introducing a little extra resistance in the galvanometer circuit, but does not at all change the balance.

Let us now consider some of the results obtained. Fig. 3 shows the complete curves of a coil during one cycle and Fig. 4 shows the portion of the curves during the break on an enlarged scale. As will be seen the current curve starts from zero, rises gradually to its maximum value, and descends rapidly to zero during the break. The duration of make was 0.029 second and that of the break, 0.0030 second. During the make, as was pre-

viously pointed out, follows the law $i = I(1 - e^{-Rt/L})$. The values of the constants were determined for this case and were as follows, $E = 3.33$, R (of entire circuit) = 1.276 and $L = 0.0422$. The curve that would be obtained using these values, differs inappreciably from the one in Fig. 3.

Near the top of the curve it bends over gradually. This is due to the fact that the brush is beginning to slide off from the contact, thus increasing the resistance. As soon as the break actually occurs, the current drops very rapidly and the curve is practically a straight line. This is seen better from Fig. 4. The fact that

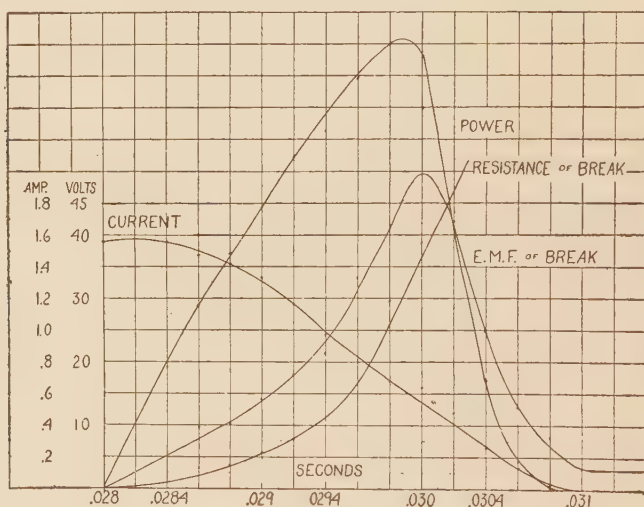


Fig. 4.

the current during the break follows practically a straight line is of importance, as this assumption was made in the theoretical part of this investigation.

The curve of E.M.F. across the break is next in importance. From the moment of contact until the break begins to take place the potential difference is very small, amounting only to the product of the current and the resistance of the contact. It was accurately measured and found to increase according to the same law as the current, as was of course to be expected. The values, however, are too small to show in the curve. At the instant of the break, the stored magnetic energy of the coil is liberated and shows itself in

the sudden and large rise in voltage. In this case the maximum voltage was 49.5 volts. After reaching its maximum it very rapidly drops to 3.33 volts, the E.M.F. of the battery, which is of course applied across the gap as long as the the circuit is open.

The curve of the E.M.F. over the coil is next to be considered. At the instant of make it jumps to approximately the E.M.F. of the battery, and then gradually decreases during the make. At the instant of break, the induced E.M.F. is suddenly reversed by the decrease of the lines of induction and a peak much like that of the Eb curve is produced, but in the opposite direction.

The last curve is that of the potential difference of the battery. It of course drops off gradually during the make as the current is increased and again rises during the break to its former value. Since the sum of all the E.M.F.'s in any closed circuit is zero, calling the E.M.F. over the non-inductive resistance Er , we should have $Eb = Er + Ebr + Ec$ each being of course taken with its proper sign. This is a good test of the correctness of the work and as is seen from the curves is very accurately fulfilled.

In Fig. 4 two additional curves are plotted those of power and resistance of break. These are computed curves, the latter being obtained by dividing Eb by i . It of course starts from almost zero, and quickly rises, becoming infinite at the instant of complete break.

The other curve W is the curve of power. It is obtained by multiplying together the instantaneous values of current and E.M.F. during the break. The maximum of this curve is 31.4 watts. This is very important as it is probable that good ignition is dependent upon the maximum of this curve rather than upon its total area.

We are now in a position to solve our main problem, the determination of the efficiency. This may be obtained either for the whole circuit including the battery or for the coil alone. The former is slightly simpler and will be here considered.

The rate of input of energy is evidently Ei where E is the E.M.F. of the battery and i is the current. This lasts during both make and break. Since E is constant, the total input is simply (average i) $\times E \times t$, where t is time of both make and break. Here average $i = 0.895$ ampere, $E = 3.33$ volts and $t = 0.0318$ second. Hence $W = 0.895 \times 3.33 \times 0.0318 = 0.0942$ joule.

Similarly the output is given by (average ei) $\times tb$, in which e is the E.M.F. across the break only. Average ei is the same as average W , or the average of the curve W . This from the curve is found to be 16.41 watts, and $tb = 0.00309$ second. Hence the output is 0.0508 joule. Finally efficiency equals

$$\frac{\text{output}}{\text{input}} = \frac{.0508}{.0942} = 54.0 \text{ per cent.}$$

A comparison of this value with that obtained according to the theory previously developed is of interest. Calculating the various quantities by the method there explained we obtain the following where all quantities are expressed in joules :

Magnetic energy,	0.0514
Energy added during break,	0.0080
Eddy current loss during make,	0.00004
Eddy current loss during break,	0.0006
Ohmic current loss during make,	0.0385
Ohmic current loss during break,	0.0032
Hysteresis loss,	0.0010

Substituting in the formula for efficiency gives

$$\eta = \frac{514 + 80 - 32 - 6}{514 + 80 + 10 + 0.4 + 385} = \frac{556}{989} = 56.2 \text{ per cent.}$$

This computed value agrees very well with the observed value of 54 per cent.

It should be pointed out that this coil was not operating under the best conditions. The energy in the spark, 0.0508 joule, was about three times as much as is needed for good ignition. Hence it would have been practicable to cut down very greatly the time of make. This would have reduced the ohmic loss of the coil, which as may be seen above is much greater than any of the others. With a contact of about $\frac{1}{100}$ second the efficiency of the coil as was shown before is about 85 per cent.

Fig. 5 shows curves for another coil. This coil was much larger than the first one, contained more iron, and consequently had an inductance about twice as great. The resistance, however, was only slightly greater. The speed of the rotating break was increased so that the make was about two thirds as long as before. The influence of the inductance is clearly seen in the much slower rise of the

current. The voltage at break on account of the high inductance and the quick break rises higher than before reaching 67 volts.

At the right are given on an enlarged scale the curves of volts, watts, amperes and ohms at break. These show the same general shapes as before. The input of the coil is of course less and the efficiency works out to be 43.2 per cent.

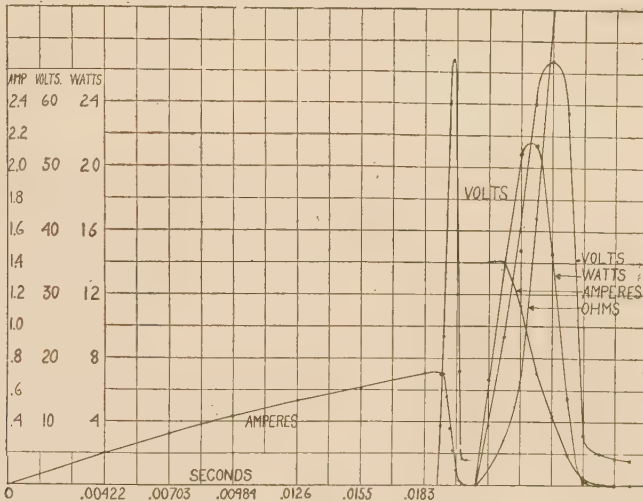


Fig. 5.

In Fig. 6 we have on the other hand the curves of a coil having an inductance of only 0.0121 henry; about one fourth of that of the coil of Fig. 3 and about one seventh of that of Fig. 5. This was operated with a voltage of only 2.072 that of one storage cell; in fact the coil was wound to test the possibility of operating a primary spark coil with a very low voltage. The time of make and break are about the same as in Fig. 4. The current rises very rapidly on account of the low inductance. The voltage at break, however, is only about half as great as in the other two cases, but the current is so much larger that the maximum watts and hence presumably the igniting power is about the same. The efficiency is slightly higher, being 49.5 per cent.

THE INDUCTION COIL OR SECONDARY COIL.

In studying the induction coil we shall first consider the simple case of a coil with the secondary open and no condenser used in

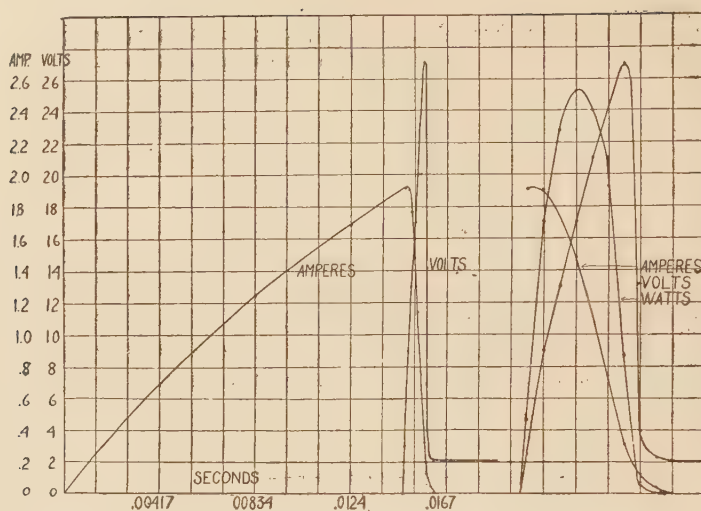


Fig. 6.

the primary. Under these conditions, the electrical actions are in all respects similar to those of the primary coil. To be sure the secondary winding has a certain capacity, and the E.M.F. induced in it causes a slight displacement of electricity. This effect is greatest in large coils wound with very fine wire, but even there it is entirely negligible. With coils for very high frequency, say in the neighborhood of 100,000 cycles per second, the case is entirely different, and this capacity may have a very decided effect upon the voltage produced. For the present we shall consider only coils in which the frequency does not exceed 1,000 cycles per second, and this effect will consequently be neglected.

For the coil then without condenser and whose secondary is open no farther investigation is necessary. Figs. 4 and 5 show all the actions taking place with the single exception of the curve of secondary E.M.F. This is, however, readily derived. Since we have assumed that there is no current in the secondary, it will obviously have no effect upon the primary, and the primary current during make will follow the usual law $i_p = I(1 - e^{-Rt/L})$. We then have

$$e_s = M \frac{di_p}{dt} = \frac{MRI}{L} e^{-\frac{Rt}{L}} = \frac{M}{L} E e^{-\frac{Rt}{L}}.$$

This gives us the value of the secondary E.M.F. at any instant dur-

ing the make. It starts with the value ME/L at $t = 0$ and gradually decreases, becoming practically zero for large values of t .

Most commercial induction coils are provided with condensers connected across the break. These serve in general two purposes. First, they prevent largely the arcing which would otherwise occur at the break and secondly they increase in general the E.M.F. induced in the secondary. In addition they may or may not set up oscillations in the circuits. We shall now examine these functions somewhat in detail.

During the make the condenser is short circuited and hence it has obviously no effect. We may then neglect this part of the cycle and concern ourselves with the break only. In general the current during the break becomes oscillatory, and dies out gradually. If the break be instantaneous we have the following equations in which all the quantities refer to the primary.

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E$$

where q is the quantity of electricity in the condenser, E is the E.M.F. of the battery and C is the capacity of the condenser. dq/dt is the current i and the equation is the same as before except for the term q/C . This is the potential difference over C the condenser. The arrangement of the circuit is shown in Fig. 7.

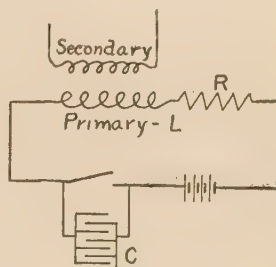


Fig. 7.

The solution of the above well-known differential equation is

$$i = \frac{dq}{dt} = Qe^{at} [\omega \cos \omega t + \alpha \sin \omega t]$$

where

$$\alpha = -\frac{R}{2L}, \quad \omega = \sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}},$$

and the time of one oscillation is given by

$$T = \frac{1}{n} = \frac{2\pi}{\sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}}.$$

Usually in practice $R^2/4L^2$ is very small compared with $1/CL$ and may be neglected. In this case we have $T = 2\pi\sqrt{LC}$. In practice ω is usually of about the order 10^3 while α is purposely kept low. Hence we may usually neglect the last term and we get $i = Q\omega\epsilon^{at} \cos \omega t$. It is evident that just at the instant of break $i = i_b$ and since $t = 0$ we have $i = Q\omega$ or substituting,

$$i = i_b \epsilon^{-\frac{Rt}{2L}} \cos \frac{t}{\sqrt{LC}}.$$

This means that the current oscillates with a gradually decreasing amplitude. In fact it dies out gradually in just the same way that it grows to its final value during make, but only half as fast.

The primary E.M.F. is given by $L \cdot d^2q/dt^2$ as shown above and the secondary E.M.F. by $M \cdot d^2q/dt^2$. If as before α is negligible compared to ω we have $e_p = LQ\omega^2\epsilon^{at} \sin \omega t$ and $e_s = -MQ\omega^2\epsilon^{at} \sin \omega t$. That is in case the decrement is small the E.M.F. and the current differ 90° in phase.

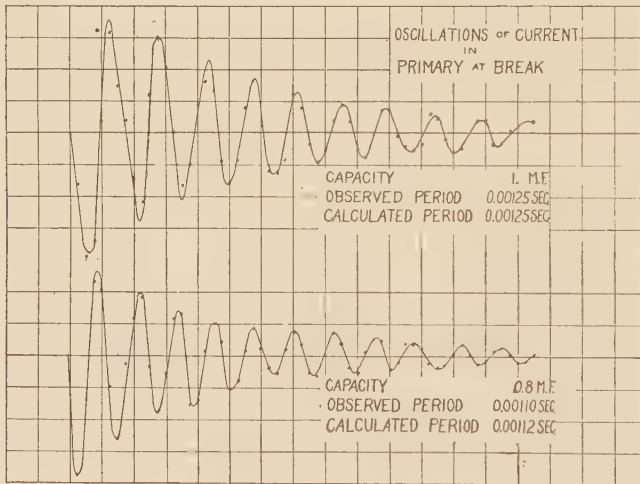


Fig. 8.

The first point taken up experimentally was a comparison of the calculated time of oscillation with the observed time. Fig. 8 for example shows a series of points on the curve of primary current of a coil having an inductance of 0.0422 henry, used with a condenser

of 1 microfarad capacity. Points could be readily taken for a long distance on the curve as shown. The number that could be obtained was, however, limited by the rotating break closing the circuit again. The points were taken one degree apart and since the contact maker was revolving 393 times per minute this corresponded to $60/393 \times 720 = 0.000213$ second. The average length of a wave in Fig. 9 is 5.88 degrees or 0.00125 second or 8,000 complete vibrations per second. The resistance of the circuit was 1.28 ohms, and substituting in the formula we get as the calculated period 0.00125 second which is the same result as before.

The lower curve of Fig. 8 is a curve of primary current for the same coil but with a smaller capacity. The two curves of Fig. 9

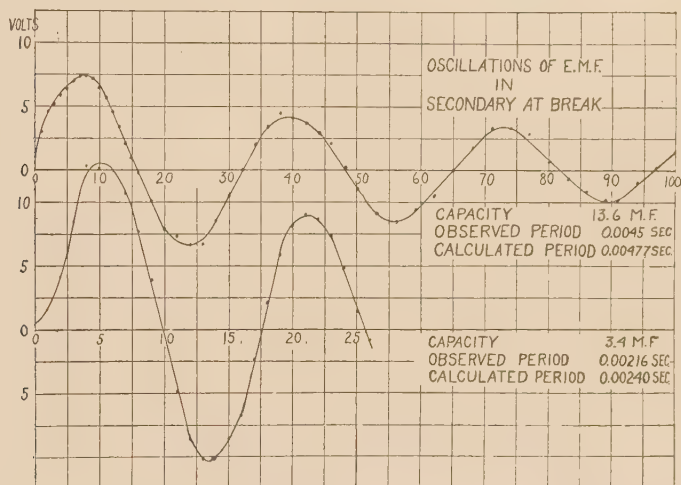


Fig. 9.

are curves of secondary E.M.F. in the same coil. The frequency of oscillation is of course the same in the secondary as in the primary and the calculated and observed times show good agreement. These four curves with the exception of the lower one of Fig. 9 are all plotted to the same scale of time and illustrate well the great variation in frequency caused by the condenser.

In the remaining curves, in order to save time, only a few points near the beginning of the curve were taken, then the contact maker was moved ahead a certain number of degrees, the number of

waves passed over counted, and a few more points taken. This supplied at least two crossings, and the number of waves between the crossings, and consequently all the data needed to determine the time of vibration were at hand. Table II. gives the computed

TABLE II.

Capacity in M.F.	Computed Time.	Observed Time.
1.0	.000125	.000125
.8	.000116	.000110
.6	.000100	.000093
.4	.000082	.000078

and observed times for a number of different values of the capacity. It will be noted that in every case except the first, the observed time is less than the computed. This is probably due to the fact that, especially with small values of the capacity, there is some burning of the contacts at the break. This burning and roughening of the surface causes the break to occur earlier and consequently has the effect of moving the later points farther to the left than they should go, thus causing the apparent discrepancy.

EFFECT OF CAPACITY UPON THE SECONDARY E.M.F.

At the moment of break the energy stored in the coil is $\frac{1}{2}Li_b^2$ and that in the condenser is zero. Assuming that there is no loss at the break, *i. e.*, no sparking, practically all of this is transferred to the condenser a quarter of a period later, from whence it again surges back into the coil and so continues until it is all dissipated in various losses. When the energy is stored in the condenser its value is $\frac{1}{2}E_c^2C$ and since as explained above this is practically equal to $\frac{1}{2}Li_b^2$ we may write

$$\frac{1}{2}E^2C = \frac{1}{2}Li_b^2$$

or

$$E_c = i_b \sqrt{\frac{L}{C}}.$$

But the maximum E.M.F. over the coil is the same as that over the condenser since they are directly connected together, hence we have for the primary

$$E_b = i_b \sqrt{\frac{L}{C}}.$$

If the magnetic leakage be neglected the primary and secondary E.M.F.'s are in the ratio of their respective turns or $E_s/E_p = N_s/N_p$. Hence we have as the final expression for the secondary E.M.F.

$$E_s = i_b \frac{N_s}{N_p} \sqrt{\frac{L}{C}},$$

and we see that both the primary and secondary E.M.F. vary inversely as the square root of the capacity.

On the other hand we have to consider the loss of energy in sparking at the break. It is of course evident that the E.M.F. across the break and that across the condenser are the same. The larger the condenser the lower the E.M.F. and the less the spark-

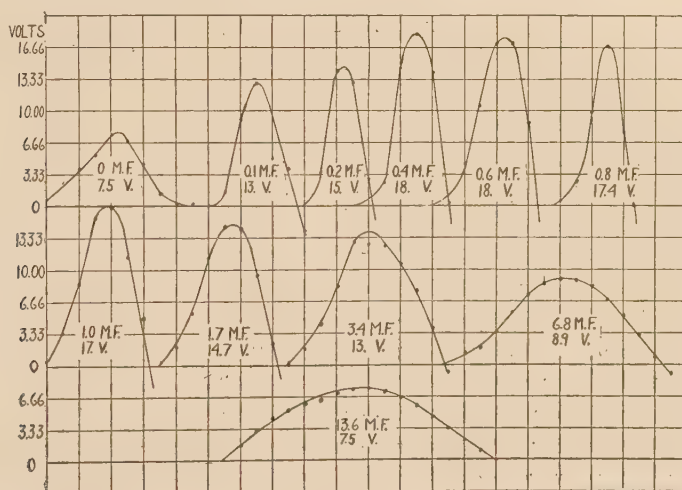


Fig. 10.

ing. It is then quite evident that we have here two conflicting elements and it appears probable at once that a compromise will give the best results. This is already known to be the case¹ and is shown very clearly in Fig. 10.

This figure shows curves of secondary E.M.F. for gradually increasing values of the capacity. With zero capacity the E.M.F. rises to 7.5 volts and gradually falls. In this case there is no oscillation. With 0.1 microfarad the E.M.F. rises to 13 volts and we

¹ Mizuno, Philosophical Mag., 5th Series, No. 45, p. 447.

get a series of oscillations in the secondary. This is also the case in all the following curves, but to save space only the first half wave is shown. This in every case gave the greatest value of the voltage. The succeeding curves show increasing values of the E.M.F. up to a capacity of 0.4 microfarad and beyond this point they decrease regularly. In this particular case then 0.4 microfarad is the best capacity if maximum secondary E.M.F. be the only consideration. If efficiency also is to be considered, a larger value of the capacity would be desirable.

It is possible by means of the formula already derived to calculate the values of the E.M.F. that should be developed in the secondary at break. This is done by means of the formula

$$E_s = \frac{N_s}{N_p} i_b \sqrt{\frac{L}{C}},$$

and it furnishes us with the means of computing the E.M.F. of any induction coil. To do this it is necessary to know the values of i_b , L , N_s , N_p and C . These can, however, be readily measured in any particular coil.

In the case of the coil used in obtaining these curves, the current at break was 1.20 amperes, and the other values are as given before. In the case of the last curve of Fig. 10 we have

$$E_s = \frac{584}{504} \times 1.20 \sqrt{\frac{.0422}{.0000136}} = 7.70 \text{ volts.}$$

The observed value as shown by the curve is 7.5 volts, which agrees very well with the calculated value.

TABLE III.

Speed of Breaking = 80 Inches per Second.

Capacity in M.F.	Maximum Observed.	Computed E.M.F.
0.0	7.5	infinity
0.1	13.0	91.2
0.2	15.0	64.4
0.4	18.0	45.5
0.6	18.0	37.2
0.8	17.4	32.2
1.0	17.0	28.1
1.7	14.7	22.1
3.4	13.0	16.5
6.8	8.9	11.0
13.6	7.5	7.8

In the same way the E.M.F.'s for the other values of the capacity were calculated. The results as well as the observed maxima of the waves are given in Table III. and the same quantities are plotted in Fig. 11. They show good agreement for the larger values of the capacity, the observed E.M.F. being always less than the computed one on account of the loss at the break. The dis-

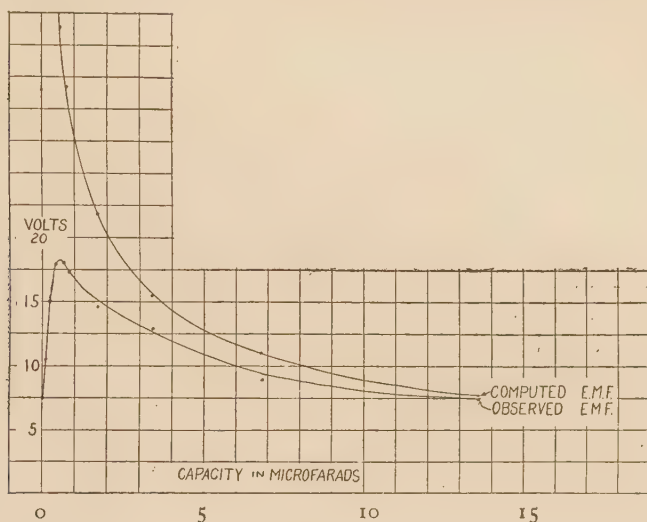


Fig. 11.

crepancy becomes greater as the capacity becomes less, on account of the condenser being less able to suppress the sparking. It is apparent from the curve that if the object be to get the highest possible secondary E.M.F., *i. e.*, the greatest spark length, the condenser should have a value of 0.4 M.F. However with this capacity there is considerable sparking and the efficiency is rather low. Consequently for general purposes a somewhat larger capacity would be preferable.

EFFECT OF CURRENT IN THE SECONDARY.

So far in these experiments, no current has been taken from the secondary, that is the coil has been on open circuit. We have now to enquire into the changes that are produced in practice, when the secondary is used to supply current. Figs. 12 and 13 were taken to show the action in this case.

The coil used had a core consisting of about 600, No. 16 soft iron wires, forming a bundle $6\frac{3}{8}$ " long by $1\frac{3}{4}$ " in diameter. The secondary and the primary each contained 252 turns of No. 13 wire. The secondary was wound outside of the primary and thus had a higher resistance. Otherwise they were alike.

This plan was adopted to keep the secondary E.M.F. within the limits of about 100 volts for the sake of ease of measurement.

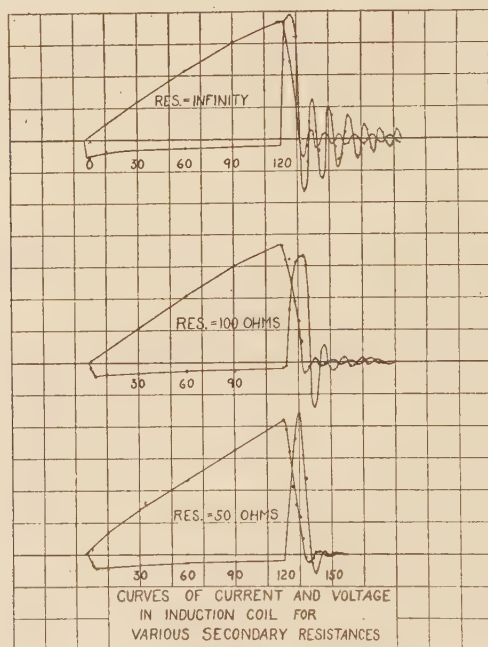


Fig. 12.

The first curve of Fig. 12 shows the variations of secondary E.M.F. and primary current in the coil with open secondary. The primary current as before rises gradually, breaks and dies away in a series of oscillations of decreasing amplitude. The secondary E.M.F. at make jumps up almost instantly to its maximum make value. This value is practically equal to the battery voltage multiplied by the ratio of the secondary to the primary turns. The equation of this curve has been obtained and is

$$e_s = \frac{M}{L} E e^{-\frac{Rt}{L}}.$$

If the leakage is small $M = \sqrt{L_p L_s}$ where L_p and L_s are the inductances of the primary and secondary respectively ; hence

$$e_s = -E \sqrt{\frac{L_s}{L_p}} e^{-\frac{Rt}{L}},$$

and since the inductance is proportional to the square of the number of turns this becomes

$$e_s = E \frac{N_s}{N_p} e^{-\frac{Rt}{L}}.$$

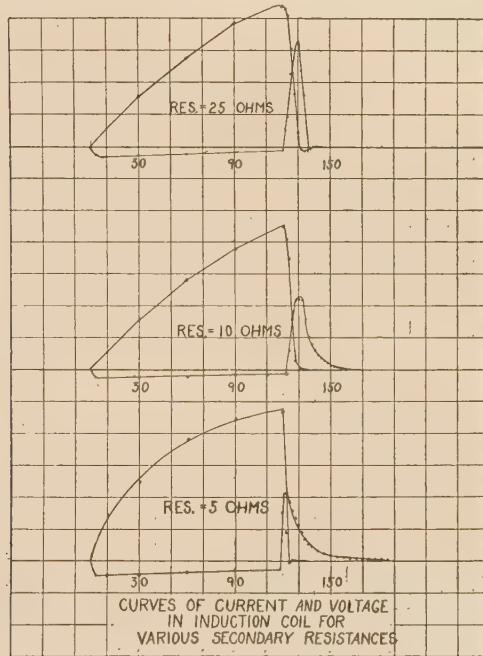


Fig. 13.

At $t = 0$ this becomes $E \cdot N_s / N_p$ and decreases gradually as t increases, in fact approaching zero according to the same law as i approaches I . The actual curve agrees closely with the theoretical.

This make E.M.F. is usually small compared with that at the break. It may be made smaller or larger by decreasing or increasing E the battery voltage. For certain purposes, notably in X-ray work it is important to keep this make E.M.F. low in order to avoid

current in the wrong direction. Here it would be well to keep E as small as possible. A coil used with a Wehnelt interrupter on a 110-volt or 220-volt circuit naturally gives much trouble from inverse current on this account. Such a coil should work much better on an alternating than on a direct current, since the current would in general not rise so rapidly.

At the break the conditions are quite different. We have shown that here we have

$$i = i_b e^{-\frac{Rt}{2L}} \cos \frac{t}{\sqrt{LC}} \quad \text{and} \quad e_s = -M i_b \frac{1}{\sqrt{LC}} e^{-\frac{Rt}{2L}} \sin \frac{t}{\sqrt{LC}}.$$

These equations show that both the primary current and the secondary E.M.F. will die out in a series of oscillations, the rate of dying out, *i. e.*, the logarithmic decrement will be half that of the current and E.M.F. at the make since here the coefficient was $e^{-Rt/L}$. Moreover the current and E.M.F. will be so related to one another that one will be zero when the other is a maximum, *i. e.*, they differ 90° in phase. All these conclusions are verified from the curves and are all obvious except the second. To show this let us examine the ratio of the maxima of the two successive E.M.F. waves. For this we may take $\omega t = 0$ and $\omega t = 2\pi$ or $t = 0$ and $t = 2\pi/\omega$; we then have

$$\text{Ratio} = \frac{-M i_b \omega e^{-R\omega/2L} \sin 0}{-M i_b \omega e^{-R\pi/L\omega} \sin 2\pi} = \frac{e^{-0}}{e^{-R\pi/L\omega}} = e^{\frac{R\pi\sqrt{CL}}{L}} = e^{R\pi\sqrt{\frac{C}{L}}}.$$

This then should be the ratio of two successive waves and substituting the values of the quantities we have $\text{Ratio} = 2.72^{.0391} = 1.04$. Hence each succeeding wave should be about 4 per cent. lower than the preceding one. The actual decrease is about 25 per cent. instead of 4 per cent. The difference is largely due to the loss in eddy currents which act just like a load on the secondary.

As soon as current is taken from the secondary the conditions change. The solution of the equations becomes quite complicated and will not be given here.¹ It is evident however, that if the secondary were connected to a condenser, the current in the second-

¹ A fairly complete treatment of these equations may be found in Fleming's "The Alternating Current Transformer," pages 183 to 191 or in an article by J. E. Ives, PHYSICAL REVIEW, vol. 14, p. 280, and vol. 15, p. 7.

dary would tend to oscillate in the same way that that in the primary does. The two currents would not necessarily have the same frequency nor the same decrement. Owing, however, to the mutual action of the primary and secondary, both currents would take up an oscillation which would be the sum of two oscillations of different frequencies and decrements. In the case of a secondary connected to a non-inductive resistance, the secondary current of itself will not oscillate.

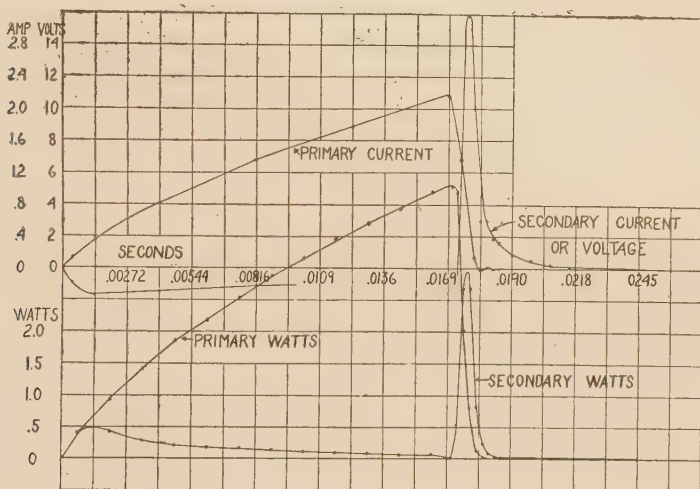


Fig. 14.

Hence, on account of the mutual induction of the two coils it will tend to prevent the primary current from oscillating. This action of the secondary current will obviously be the more powerful the less the secondary resistance, that is, the greater the secondary current.

Figs. 12 and 13 show the effect of increasing gradually the current taken from the secondary. The resistance used was non-inductive, hence the curve of terminal secondary E.M.F. may be equally well considered the curve of secondary current, since the current and E.M.F. are in phase. In all the curves we notice at the moment of make a rounding of the curve of secondary E.M.F. or current. This is of course due to the inductance of the secondary tending to prevent the current from rising too rapidly.

In the second curve of Fig. 12 with 100 ohms secondary resistance oscillations are present although much reduced and in the third curve for which the secondary resistance was 50 ohms this reduction is still more marked. In the curves of Fig. 13 with respectively 25, 10 and 5 ohms the oscillations have disappeared although a slight trace is still present in the primary current. It will be noted that the secondary E.M.F. and current attain their maximum values at about the time the primary current becomes zero. It is evident that the magnetism of the core is not zero, being maintained by the secondary current, and this magnetism now proceeds to dissipate its energy, by producing current in the secondary. In fact there is no other way for it to be dissipated, providing oscillations are not present, when it would be largely wasted in eddy currents and I^2R losses.

In the induction coil employed as it usually is to produce a spark, the curves would be somewhat different. In general no current would flow during the make, and the curve of decrease of current would be somewhat steeper, since the resistance depends upon the current flowing.

We may regard the induction coil from a slightly different standpoint. By considering only the case most frequently met with in practice, that of a non-inductive receiving circuit without capacity, the theory becomes simpler. If we further suppose that the receiving circuit has a low enough resistance so that oscillations are entirely prevented, it becomes possible to present the entire theory of the coil in a very definite and simple form, the only uncertainty being that connected with the action during the break. This period is very short compared with the duration of the break current of the secondary. The theory of the coil under these circumstances has been treated by Fleming in the book before referred to and it is there shown that during the make the currents are represented by

$$i_p = I - \frac{I}{2} \left(\varepsilon^{-\frac{Rt}{L+M}} + \varepsilon^{-\frac{Rt}{L-M}} \right)$$

and

$$i_s = -\frac{I}{2} \left(\varepsilon^{-\frac{Rt}{L+M}} - \varepsilon^{-\frac{Rt}{L-M}} \right).$$

The inductance of the primary and secondary are here considered as equal.

During the break some uncertainty exists. If, however, the break be assumed to follow a straight line law, and this seems to be amply justified by the curves taken, we have the very simple equation,

$$0 = R_s i_s + L \frac{di_s}{dt} + M \frac{di}{dt}.$$

Since we represent i as before by

$$i = i_b - \frac{t}{T} i_b, \quad M \frac{di}{dt} = - \frac{i_b}{T} M = a \text{ constant.}$$

Hence

$$M \frac{i_b}{T} = R_s i_s + L \frac{di_s}{dt},$$

is the equation of the secondary circuit. This is the ordinary Helmholtz equation and its solution is

$$i_s = \frac{Mi_b}{TR_s} \left(1 - e^{-\frac{R_s t}{L_s}} \right).$$

To find the value of i_s at the instant of complete break we have only to substitute $t = T$. We are especially interested in the case when the time of break T is very nearly zero. The expression for i_s becomes of the form $0/0$ for $T = 0$, hence writing

$$i_s = \frac{Mi_b}{R_s} \cdot \frac{(1 - e^{-R_s T/L_s})}{T}$$

and differentiating both members we get

$$\frac{\frac{R_s}{L_s} e^{-\frac{R_s T}{L_s}}}{1} \cdot \frac{Mi_b}{R_s},$$

which for $T = 0$ is equal to

$$\frac{Mi_b}{R_s} \cdot \frac{R_s}{L_s} = \frac{Mi_b}{L_s}.$$

If as is very nearly the case

$$M = \sqrt{L_s L_p} \quad \text{and} \quad \frac{L_s}{L_p} = \frac{N_s}{N_p^2}$$

we may write

$$i_{sm} = \sqrt{\frac{L_p}{L_s}} \cdot i_b = \frac{N_p}{N_s} i_b.$$

This result might have been obtained more simply by consideration of the energy involved. The work stored in the primary at the instant of break is $\frac{1}{2}L_p i_b^2$. If the break be instantaneous and hence no energy be wasted in the process, all this energy must be transferred to the secondary. We then have at once

$$\frac{1}{2}L_p i_b^2 = \frac{1}{2}L_s i_s^2 \quad \text{or} \quad i_s = \sqrt{\frac{L_p}{L_s}} \cdot i_b.$$

This is a rather surprising result at first sight, since the E.M.F. is independent of the capacity of the condenser and of the resistance in the secondary circuit. Moreover, it depends upon the inverse ratio of the turns N_p/N_s . It should be contrasted with the expression for the E.M.F. at break

$$e_{em} = \frac{N_s}{N_p} i_b \sqrt{\frac{L_p}{C}}.$$

This arises from the fact that while the E.M.F. generated is in proportion to N_s the inductance of the secondary at the same time increases in proportion to the square of N_s and hence an increase in N_s actually means a decrease of the maximum momentary current.

It is a fact that many makers of large induction coils for X-ray work are winding the secondaries with much coarser wire than formerly and the users claim that such coils pass much more current in the proper direction and much less inverse than those of the conventional design.

When the break is completed the influence of the primary upon the secondary ceases almost entirely. We have then merely to consider the dying away of a current in an inductive circuit. The equation is $0 = R_s i_s + L_s di_s/dt$. The well-known solution of this is $i_s = i_{sm} e^{-R_s t/L_s}$. Theoretically this would require an infinite time for the current to die away. As may be seen by Figs. 13 and 14 the actual time was comparatively long, and traces of current could be detected for some time beyond that shown.

The curves of Figs. 13 and 14 confirm the preceding theory very closely, with the possible exception of that relating to the current during break. The E.M.F. generated during this time is so great and the consequent rise of current is so rapid that it is difficult to follow it. However, as nearly as can be seen from the curves, the shape is apparently that called for by the theory.

EFFICIENCY.

The efficiency of an induction coil may be obtained in the same way as in the case of the primary coil. An examination of the curves of Figs. 11 and 12 indicates that the maximum secondary output will be obtained with some value of the secondary resistance between 5 and 10 ohms. It was estimated that 8 ohms would probably be the resistance giving the greatest output and the curves of Fig. 13 were taken with this value. The upper curves, those of primary current and secondary voltage, were taken directly. The curves of watts in the primary and secondary were computed from the readings, the points of the secondary watts being given by E_s^2/R_s . This latter would not be possible had R_s not been non-inductive. It should be noted that the secondary watts at the break are plotted on a scale only one tenth as great as is used in the other curves. It is interesting to note that just after the make the curve of secondary watts rises almost as fast as that of primary watts, or the output is almost as great as the input. Hence the efficiency is high and the relative amount of energy delivered during the make is great. If it is desirable to get as much energy as possible from the coil during the make, the contact should be short and the battery voltage high. The usual requirement is just the reverse of this, as was pointed out, but for medical coils, such a distribution of energy between the make and break is frequently preferable.

The efficiency is readily obtained from the curve by dividing the area under the curve of secondary watts by that under that of primary watts. This gives here the following results :

Input during make,	0.0417 joule
Input during break,	0.0032 joule
Output during make,	0.0031 joule
Output during break,	0.0193 joule
Total input,	0.0449 joule
Total output,	0.0224 joule

The efficiency is then .0224/.0449 or 50 per cent.

If the coil were used to produce a spark and the gap resistance were high enough so that no current passed during the make, the output would be approximately 0.0193 joule. The input would

not be greatly affected and the resulting efficiency would be 0.0193/0.0449 or 43 per cent.

In any given coil it is possible to predetermine, to a certain extent the output, at least we can set an upper limit to the energy of each spark, and thus compare the igniting power of various coils. This follows from the fact that the energy in the spark cannot possibly exceed the stored magnetic energy of the coil. That is, it cannot exceed $\frac{1}{2}Li_b^2$. L and i_b are determined in the manner previously explained. For example in the present case

$$\frac{1}{2}Li_b^2 = \frac{1}{2} \times 0.0121 \times 2.18^2 = 0.0288 \text{ joule.}$$

The measured output was 0.0193 joule or about 67 per cent. of the possible output.

On the other hand the input during make will of course be larger than the stored magnetic energy. In this case they are in the ratio of 0.0288 to 0.0417 or of the energy put in during wake, about 70 per cent. is stored as magnetic energy.

To recapitulate, by measuring for any coil the five quantities R , L , C , i_b , N_s/N_p , or the resistance, capacity, current at break and ratio of primary to secondary turns we can determine the four principal characteristics of the coil. Its secondary E.M.F. is given by

$$E_s = i_b \frac{N_s}{N_p} \sqrt{\frac{L}{C}},$$

its maximum output per spark by $\frac{1}{2}Li_b^2$, its frequency of oscillation by $n = 2\pi\sqrt{LC}$ and the maximum current at break by $i_{sm} = i_b \cdot N_p/N_s$.

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